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Masses and couplings of the lightest Higgs bosons in $(M+1)$SSM

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Abstract. We study the upper limits on the mass of the lightest and second lightest CP even Higgs bosons in the $(M+1)$SSM, the MSSM extended by a gauge singlet. The dominant two loop contributions to the effective potential are included, which reduce the Higgs masses by $\sim 10\,\text{GeV}$. Since the coupling R of the lightest Higgs scalar to gauge bosons can be small, we study in detail the relations between the masses and couplings of both lightest scalars. We present upper bounds on the mass of a "strongly" coupled Higgs ($R > 1/2$) as a function of lower experimental limits on the mass of a "weakly" coupled Higgs ($R < 1/2$). With the help of these results, the whole parameter space of the model can be covered by Higgs boson searches.

1 Introduction

Curiously enough, the most model independent prediction of supersymmetric extensions of the standard model concerns a "standard" particle: the mass m_h of the (lightest CP even) Higgs boson. Within the minimal supersymmetric extension of the standard model (MSSM) its mass is bounded, at tree level, by

$$m_h^2 \leq M_Z^2 \cos^2 2\beta, \tag{1.1}$$

where $\tan\beta = h_1/h_2$ (H_1 couples to up-type quarks in our convention). It has been realized already some time ago that loop corrections weaken this upper bound [1]. These loop corrections depend on the top quark Yukawa coupling h_t and the soft SUSY breaking parameters as the stop masses of $O(M_{\text{SUSY}})$. At the one loop level, given the present experimental errors on the top mass m_t and assuming $M_{\text{SUSY}} \lesssim 1\,\text{TeV}$, the upper limit on m_h is $\lesssim 140\,\text{GeV}$. Also two loop corrections to m_h have been considered in the MSSM [2–4]; these have the tendency to lower the upper bound on m_h by $\sim 10\,\text{GeV}$.

The subject of the present paper is the next-to-minimal supersymmetric extension of the standard model ($(M+1)$SSM) [5–13] where a gauge singlet superfield S is added to the Higgs sector. It allows one to omit the so-called μ term $\mu H_1 H_2$ in the superpotential of the MSSM, and to replace it by a Yukawa coupling (plus a singlet self-coupling):

$$W = \lambda S H_1 H_2 + \frac{\kappa}{3} S^3 + \ldots \tag{1.2}$$

The superpotential (1.2) is thus scale invariant, and the electroweak scale appears only through the SUSY breaking terms.

In view of ongoing Higgs searches at LEP2 [14–16] and, in the near future, at Tevatron Run II [17], it is important to check the model dependence of bounds on the Higgs mass. In the $(M+1)$SSM, the upper bound on the mass m_1 of the lightest CP even Higgs[1] differs from the one of the MSSM already at tree level: now we have [5,6]

$$m_1^2 \leq M_Z^2 \left(\cos^2 2\beta + \frac{2\lambda^2}{g_1^2 + g_2^2} \sin^2 2\beta \right), \tag{1.3}$$

where g_1 and g_2 denote the $U(1)_Y$ and the $SU(2)_{\text{L}}$ gauge couplings. Note that, for $\lambda < .53$, m_1 is still bounded by M_Z at tree level. Large values of λ, $\lambda > .7$, are in any case prohibited, if one requires the absence of a Landau singularity for λ below the GUT scale [5,6].

Loop corrections to m_1 have also been considered in the $(M+1)$SSM [6]. Given m_t and assuming again $M_{\text{SUSY}} < 1\,\text{TeV}$, the upper limit on m_1 at one loop is then $\sim 150\,\text{GeV}$. Within the constrained $(M+1)$SSM (the C$(M+1)$SSM), where universal soft SUSY breaking terms at the GUT scale are assumed [7–10], λ is always below $\sim .3$, and the upper limit on m_1 reduces to the one of the MSSM (at one loop) of $\sim 140\,\text{GeV}$. Two loop corrections in the $(M+1)$SSM have recently been considered in [13].

Within the $(M+1)$SSM this is, however, not the end of the story: It is well known [7,10,11] that now the lightest Higgs scalar S_1 can be dominantly a gauge singlet state. In

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[1] As there are three CP even Higgs states in the $(M+1)$SSM, we denote them as S_i with masses m_i, $i = 1, \ldots, 3$, in increasing order

this case it decouples from the gauge bosons and becomes invisible in Higgs production processes, and the lightest *visible* Higgs boson is then actually the second lightest one S_2. Fortunately, under these circumstances S_2 cannot be too heavy [7,10,11]: In the extreme case of a pure singlet lightest Higgs, the mass m_2 of the next-to-lightest Higgs scalar is again below the upper limit designed originally for m_1. In general, however, mixed scenarios can be realized, with a weakly coupled (but not pure singlet) lightest Higgs, and a second lightest Higgs above the previous m_1 limits. Although analyses of the Higgs sector including these scenarios in the $(M+1)$SSM have been presented before [11] we find that these should be improved: First, experimental errors on the top quark pole mass $m_t^{\text{pole}} = 173.8 \pm 5.2\,\text{GeV}$ [18] have been reduced considerably, leading to stronger constraints on the top quark Yukawa coupling h_t which determines to a large extent the radiative corrections to $m_{1,2}$. Second, at least the dominant two loop corrections to the effective potential should be taken into account, since they are not necessarily negligible. The purpose of the present paper is thus an analysis of the allowed masses and couplings to the gauge bosons of the lightest CP even Higgs scalars in the $(M+1)$SSM, including present constraints on m_t and a two loop improvement of the Higgs potential.

In the next section we present our method of obtaining the dominant two loop terms in the effective potential, and in Sect. 3 we give the resulting upper bound on the lightest Higgs mass. Albeit that this upper limit can be obtained analytically, the mass of the second lightest Higgs in relation to the coupling to the gauge bosons requires a numerical analysis. Our methods of scanning the parameter space of the model in two different scenarios (constrained and general $(M+1)$SSM) are presented in Sect. 4. Results on the Higgs masses and couplings and conclusions are presented in Sect. 5.

2 Two loop corrections

In order to obtain the correct upper limit on the Higgs boson mass in the presence of soft SUSY breaking terms, radiative corrections to several terms in the effective action have to be considered. Let us first introduce a scale $Q \sim M_{\text{SUSY}}$, where M_{SUSY} is of the order of the SUSY breaking terms. Let us assume that quantum corrections involving momenta $p^2 \gtrsim Q^2$ have been evaluated; the resulting effective action $\Gamma_{\text{eff}}(Q)$ is then still of the standard supersymmetric form plus soft SUSY breaking terms. Assuming correctly normalized kinetic terms (after appropriate rescaling of the fields), the Q dependence of the parameters in $\Gamma_{\text{eff}}(Q)$ is given by the supersymmetric β functions (valid up to a possible GUT scale M_{GUT}).

Often one is interested in relating the parameters in $\Gamma_{\text{eff}}(Q)$ to more fundamental parameters at M_{GUT}. To this end one integrates the supersymmetric renormalization group equations between M_{GUT} and $Q \sim M_{\text{SUSY}}$ to one or, if one whishes, to two loop accuracy. Note, however, that the limits on the Higgs boson mass depend exclusively on the parameters in $\Gamma_{\text{eff}}(Q)$ at the scale $Q \sim M_{\text{SUSY}}$; the two loop contributions to the effective potential considered below serve to specify this dependence more precisely. The accuracy to which one has (possibly) related the parameters at the scale $Q \sim M_{\text{SUSY}}$ to parameters at a scale M_{GUT} is completely irrelevant for the relation between the Higgs boson mass and the parameters at the scale $Q \sim M_{\text{SUSY}}$.

One is left with the computation of quantum corrections to Γ_{eff} involving momenta $p^2 \lesssim Q^2$. Subsequently the quantum corrections to the following terms in Γ_{eff} will play a role.

(a) Corrections to the kinetic terms of the Higgs bosons. Due to gauge invariance the same quantum corrections contribute to the kinetic energy and to the Higgs–Z boson couplings, which affect the relation between the Higgs VEVs and M_Z.
(b) Corrections to the Higgs–top quark Yukawa coupling.
(c) Corrections to the Higgs effective potential. These corrections could, in principle, be decomposed into contributions to the Yukawa couplings λ and κ of (1.2) and the soft terms (these contributions are the ones proportional to $\ln Q^2$ or, at two loop order, $\ln^2 Q^2$), and "non-supersymmetric" contributions which are Q^2 independent. These latter contributions to the effective potential are of the orders $(\text{VEV})^n$ with $n > 4$ and become small in the case of large soft terms compared to the VEVs. Our results in Sect. 5 are based on the effective potential *including* these contributions (which are not necessarily numerically irrelevant), and there is no need to perform the decomposition of the radiative corrections to the effective potential explicitly.

Let us start with the last item: The Higgs effective potential V_{eff} can be developed in power of $\hbar$ or loops as

$$V_{\text{eff}} = V^{(0)} + V^{(1)} + V^{(2)} + \ldots \tag{2.1}$$

Within $(M+1)$SSM, we are interested in the dependence of V_{eff} in three CP even scalar VEVs h_1, h_2 and s (assuming no CP violation in the Higgs sector). The tree level potential $V^{(0)}$ is determined by the superpotential (1.2) and the standard soft SUSY breaking terms [5–11]. For completeness, and in order to fix our conventions, we give here the expression for $V^{(0)}$:

$$\begin{aligned} V^{(0)} &= m_{H_1}^2 h_1^2 + m_{H_2}^2 h_2^2 + m_S^2 s^2 - 2\lambda A_\lambda h_1 h_2 s + \frac{2}{3}\kappa A_\kappa s^3 \\ &\quad + \lambda^2 h_1^2 h_2^2 + \lambda^2 (h_1^2 + h_2^2) s^2 - 2\kappa\lambda h_1 h_2 s^2 + \kappa^2 s^4 \\ &\quad + \frac{g_1^2 + g_2^2}{8} (h_1^2 - h_2^2)^2. \end{aligned} \tag{2.2}$$

The one loop corrections to the effective potential are given by

$$V^{(1)} = \frac{1}{64\pi^2} \text{STr} M^4 \left[\ln\left(\frac{M^2}{Q^2}\right) - \frac{3}{2} \right], \tag{2.3}$$

where we only take top and stop loops into account. The relevant field dependent masses are the top quark mass

$$m_t = h_t h_1, \tag{2.4}$$

and the stop mass matrix (in the $(T_{\mathrm{R}}^{\mathrm{c}}, T_{\mathrm{L}})$ basis)

$$\begin{pmatrix} m_T^2 + m_t^2 & m_t \widetilde{A}_t \\ m_t \widetilde{A}_t & m_Q^2 + m_t^2 \end{pmatrix}, \tag{2.5}$$

where m_T, m_Q are the stop soft masses and

$$\widetilde{A}_t = A_t - \lambda s \cot\beta \tag{2.6}$$

is the so-called stop mixing. In (2.5) we have neglected the electroweak D terms which would only give small contributions to the effective potential in the relevant region $m_T, m_Q \gg M_Z$. The masses of the physical eigenstates $\widetilde{t}_1, \widetilde{t}_2$ then read

$$m_{\widetilde{t}_1,\widetilde{t}_2}^2 = M_{\mathrm{SUSY}}^2 + m_t^2 \pm \sqrt{\delta^2 M_{\mathrm{SUSY}}^4 + m_t^2 \widetilde{A}_t^2}. \tag{2.7}$$

with

$$M_{\mathrm{SUSY}}^2 \equiv \frac{1}{2}(m_Q^2 + m_T^2)$$

and

$$\delta \equiv \left| \frac{m_Q^2 - m_T^2}{m_Q^2 + m_T^2} \right|. \tag{2.8}$$

Note that the top Yukawa coupling h_t in (2.4) and below is defined at the scale Q; cf. the discussion at the beginning of this section.

In the case of large SUSY breaking terms compared to the VEVs h_i, $V^{(1)}$ can be expanded in (even) powers of h_i. The terms quadratic in h_i will not affect the upper bound on the Higgs mass (and can be absorbed into the unknown soft parameters m_{H_1}, m_{H_2} and A_λ in (2.2)). In the approximation where the stop mass splitting δ is small[2], the quartic terms read

$$V^{(1)}\Big|_{h_i^4} = \frac{3h_t^4}{16\pi^2} h_1^4 \left(\frac{1}{2}\widetilde{X}_t + t \right), \tag{2.9}$$

where

$$t \equiv \ln\left(\frac{M_{\mathrm{SUSY}}^2 + m_t^2}{m_t^2} \right), \tag{2.10}$$

and

$$\widetilde{X}_t \equiv 2 \frac{\widetilde{A}_t^2}{M_{\mathrm{SUSY}}^2 + m_t^2} \left(1 - \frac{\widetilde{A}_t^2}{12(M_{\mathrm{SUSY}}^2 + m_t^2)} \right). \tag{2.11}$$

In our computations, however, we used the full expression (2.3) for $V^{(1)}$; we will use the quartic terms (2.9) in the next section only in order to compare our two loop result to those of [4,13].

[2] This approximation is well motivated in the C$(M+1)$SSM where we take universal soft terms at the GUT scale. On the other hand, we have checked numerically that, in general $(M+1)$SSM, the lightest Higgs mass takes its maximal value for $\delta \sim 0$

Next, we consider the dominant two loop corrections. These will be numerically important only for large SUSY breaking terms compared to h_i; hence we will expand again in powers of h_i. Since the terms quadratic in h_i can again be absorbed into the tree level soft terms, we just consider the quartic terms, and here only those which are proportional to large couplings: terms $\sim \alpha_{\mathrm{s}} h_t^4$ and $\sim h_t^6$. Finally, we are only interested in leading logs (terms quadratic in t). The corresponding expression for $V^{(2)}$ can be obtained from the explicit two loop calculation of V_{eff} in [3] or, as we have checked explicitly, from the requirement that the complete effective potential has to satisfy the renormalization group equations also at scales $Q < M_{\mathrm{SUSY}}$, provided the non-supersymmetric β function for h_t is used. One obtains in both cases

$$V_{\mathrm{LL}}^{(2)} = 3 \left(\frac{h_t^2}{16\pi^2} \right)^2 h_1^4 \left(32\pi\alpha_{\mathrm{s}} - \frac{3}{2} h_t^2 \right) t^2. \tag{2.12}$$

Now, we turn to the quantum corrections to the Higgs boson kinetic terms. They lead to a wave function renormalization factor Z_{H_1} in front of the $D_\mu H_1 D^\mu H_1$ term with, to order h_t^2,

$$Z_{H_1} = 1 + 3\frac{h_t^2}{16\pi^2} t. \tag{2.13}$$

Finally, the quantum corrections to the H_1–top quark Yukawa coupling h_t have to be considered. After an appropriate rescaling of the H_1 and top quark fields in order to render their kinetic terms properly normalized, these quantum corrections lead to an effective coupling $h_t(m_t)$ with, to orders h_t^2, α_{s},

$$h_t(m_t) = h_t(Q) \left(1 + \frac{1}{32\pi^2} \left(32\pi\alpha_{\mathrm{s}} - \frac{9}{2} h_t^2 \right) t \right). \tag{2.14}$$

In (2.13) and (2.14) the large logarithm t is actually given by $\ln\left(Q^2/m_t^2\right)$ where Q^2 acts as a UV cutoff; cf. the discussion at the beginning of this section. In the relevant region $M_{\mathrm{SUSY}} \gg m_t$ the expression (2.10) for t can be used here as well. The (running) top quark mass is then given by

$$m_t(m_t) = h_t(m_t) Z_{H_1}^{1/2} h_1, \tag{2.15}$$

and the relation between the pole and running mass, to order α_{s}, reads

$$m_t^{\mathrm{pole}} = m_t(m_t) \left(1 + \frac{4\alpha_{\mathrm{s}}}{3\pi} \right). \tag{2.16}$$

3 Upper bound on the lightest Higgs mass

In this section we derive an analytic upper bound on the mass of the lightest Higgs scalar. First, we summarize our contributions to the effective potential. As is already known, in $(M+1)$SSM the upper bound on the lightest Higgs mass m_1 is saturated when its singlet component vanishes [7,10,11,13]. One is then only interested in the h_i-dependent part of the effective potential. Assuming $h_i \ll M_{\mathrm{SUSY}}$, i.e. up to $O(h_i^4)$, one obtains from (2.2), (2.9) and (2.12)

$$\begin{aligned} & V_{\text{eff}}(h_1, h_2) \\ & = \widetilde{m}_1^2 h_1^2 + \widetilde{m}_2^2 h_2^2 - \widetilde{m}_3^2 h_1 h_2 + \frac{g_1^2 + g_2^2}{8}(h_1^2 - h_2^2)^2 \\ & + \lambda^2 h_1^2 h_2^2 + \frac{3h_t^2}{16\pi^2} h_1^4 \left(\frac{1}{2}\widetilde{X}_t + t\right) \\ & + 3\left(\frac{h_t^2}{16\pi^2}\right)^2 h_1^4 \left(32\pi\alpha_{\mathrm{s}} - \frac{3}{2}h_t^2\right) t^2, \end{aligned} \tag{3.1}$$

with

$$\begin{aligned} \widetilde{m}_1^2 &= m_{H_1}^2 + \lambda^2 s^2 + \text{rad. corrs.}, \\ \widetilde{m}_2^2 &= m_{H_2}^2 + \lambda^2 s^2 + \text{rad. corrs.}, \\ \widetilde{m}_3^2 &= 2\lambda s(A_\lambda + \kappa s) + \text{rad. corrs.} \end{aligned} \tag{3.2}$$

The radiative corrections in (3.3) stem from the contributions to $V^{(1)}$ and $V^{(2)}$ quadratic in h_i. In the large $\tan\beta$ regime (which saturates the upper bound on the lightest Higgs in MSSM), one is left with only one non-singlet light Higgs h_1, and (3.1) simplifies to

$$V_{\text{eff}}(h_1) = \widetilde{m}_1^2 h_1^2 + \widetilde{\lambda} h_1^4, \tag{3.3}$$

with

$$\begin{aligned} \widetilde{\lambda} = & \frac{g_1^2 + g_2^2}{8} + \frac{3h_t^2}{16\pi^2}\left(\frac{1}{2}\widetilde{X}_t + t\right) \\ & + 3\left(\frac{h_t^2}{16\pi^2}\right)^2 \left(32\pi\alpha_{\mathrm{s}} - \frac{3}{2}h_t^2\right) t^2. \end{aligned} \tag{3.4}$$

(Note that in the large $\tan\beta$ regime $\widetilde{A}_t = A_t$ and no dependence on the $(M+1)$SSM coupling λ is left in $\widetilde{\lambda}$.) Now, we can change the variable h_1 and replace it by a variable h_1' in terms of which the kinetic term is properly normalized, so that we have

$$M_Z^2 = \frac{g_1^2 + g_2^2}{2} h_1'^2. \tag{3.5}$$

From (2.13) one finds

$$h_1^2 \simeq h_1'^2 \left(1 - \frac{3h_t^2}{16\pi^2} t\right). \tag{3.6}$$

In terms of h_1' the effective potential reads

$$V_{\text{eff}}(h_1') = \widetilde{m}_1'^2 h_1'^2 + \widetilde{\lambda}' h_1'^4, \tag{3.7}$$

with

$$\widetilde{m}_1'^2 = \widetilde{m}_1^2 \left(1 - \frac{3h_t^2}{16\pi^2} t\right), \quad \widetilde{\lambda}' = \widetilde{\lambda}\left(1 - \frac{3h_t^2}{16\pi^2} t\right)^2. \tag{3.8}$$

Second, recall that h_t in the one loop contribution to (3.1) is given by the Yukawa coupling at the scale Q. Hence, we can replace $h_t (\equiv h_t(Q))$ in $\widetilde{\lambda}'$ by $h_t(m_t)$ using (2.14), which allows to relate it directly to the running top quark mass. Equation (2.15) now reads $m_t(m_t) = h_t(m_t) h_1'$.

From (3.7), one obtains the mass m_h of the lightest non-singlet Higgs in the case where the singlet decouples (and in the large $\tan\beta$ regime)

$$m_h^2 = \frac{1}{2} \frac{\mathrm{d}^2 V_{\text{eff}}}{\mathrm{d}h_1'^2}\bigg|_{\min} = 4\widetilde{\lambda}' h_1'^2 \bigg|_{\min}. \tag{3.9}$$

This is just the correct running Higgs mass, but does not include the pole mass corrections, which involve no large logarithms and which we will neglect throughout this paper. Using (3.5) and expanding $\widetilde{\lambda}'$ to the appropriate powers of t, the expression for m_h^2 becomes[3]

$$\begin{aligned} m_h^2 = & M_Z^2 \left(1 - \frac{3h_t^2}{8\pi^2} t\right) \\ & + \frac{3h_t^2(m_t)}{4\pi^2} m_t^2(m_t) \\ & \times \left(\frac{1}{2}\widetilde{X}_t + t + \frac{1}{16\pi^2}\left(\frac{3}{2}h_t^2 - 32\pi\alpha_{\mathrm{s}}\right)(\widetilde{X}_t + t)t\right). \end{aligned} \tag{3.10}$$

which agrees with the MSSM result in [4]. (Note, however, that the coefficient of the term $\sim \widetilde{X}_t t$ on the right hand side of (3.11) is not necessarily correct, since we would obtain terms of the same order if we would take into account simple logarithms in the two loop correction $V^{(2)}$ to the potential.)

The same procedure can be applied for general values of $\tan\beta$. Then, one has to consider the 2×2 mass matrix $(1/2)(\partial_{h_i}\partial_{h_j} V_{\text{eff}}), i, j = 1, 2$, where the h_i are properly normalized. Its smallest eigenvalue gives the following upper bound on the mass m_1 of the lightest Higgs boson for arbitrary mixings among the three states (h_1, h_2, s) [13] (which can be saturated if the lightest Higgs boson has a vanishing singlet component)

$$\begin{aligned} m_1^2 \le & M_Z^2 \left(\cos^2 2\beta + \frac{2\lambda^2}{g_1^2 + g_2^2}\sin^2 2\beta\right)\left(1 - \frac{3h_t^2}{8\pi^2} t\right) \\ & + \frac{3h_t^2(m_t)}{4\pi^2} m_t^2(m_t) \sin^2\beta \\ & \times \left(\frac{1}{2}\widetilde{X}_t + t + \frac{1}{16\pi^2}\left(\frac{3}{2}h_t^2 - 32\pi\alpha_{\mathrm{s}}\right)(\widetilde{X}_t + t)t\right). \end{aligned} \tag{3.11}$$

The only difference between the MSSM bound [4] and (3.11) is the "tree level" term $\sim \lambda^2 \sin^2 2\beta$. This term is important for moderate values of $\tan\beta$. Hence, the maximum of the lightest Higgs mass in the $(M+1)$SSM is not obtained for large $\tan\beta$ as in the MSSM, but rather for moderate $\tan\beta$ (as confirmed by our numerical analysis, cf. Sect. 5). On the other hand, the radiative corrections are identical in the $(M+1)$SSM and in the MSSM. In particular, the linear dependence in $\widetilde{X}_t$ is the same in both models. Hence, from (2.11), the upper bound on m_1^2 is

[3] In (3.11) and below in (3.11) we omit the argument of h_t wherever its choice corresponds to a higher order effect

maximized for $\widetilde{X}_t = 6$ (corresponding to $\widetilde{A}_t = 6^{1/2} M_{\rm SUSY}$, the "maximal mixing" case), and minimized for $\widetilde{X}_t = 0$ (corresponding to $\widetilde{A}_t = 0$, the "no mixing" case).

4 Parametrization of the $(M+1)$SSM

Equation (3.11) gives an upper bound on the lightest Higgs mass m_1 *regardless of its coupling to the gauge bosons*. In the extreme case of a pure singlet lightest Higgs, the next-to-lightest Higgs is non-singlet and the upper bound (3.11) actually applies to m_2. On the other hand, it can occur that the lightest Higgs is weakly coupled to gauge bosons (without being a pure singlet) and m_2 is above the limit (3.11). This case requires a numerical analysis, which will be performed in the next section. First, let us present our methods of scanning the parameter space of the $(M+1)$SSM.

Not counting the known gauge couplings, the parameters of the model are

$$\lambda\, ,\ \kappa\, ,\ h_t\, ,\ A_\lambda\, ,\ A_\kappa\, ,\ A_t\, ,\ m_{H_1}^2\, ,\ m_{H_2}^2\, ,\ m_S^2\, ,\ m_Q^2\, ,\ m_T^2, \tag{4.1}$$

where h_t is eventually fixed by the top mass and an overall scale of the dimensionful parameters by the Z mass. Now, let us see how to handle this high dimensional parameter space in two different scenarios.

4.1 Constrained $(M+1)$SSM

In C$(M+1)$SSM the soft terms are assumed universal at the GUT scale, the global minimum of the effective potential has to be the global minimum and present experimental constraints on the sparticle and Higgs masses are applied. The free parameters can be chosen as the GUT scale dimensionless parameters

$$\lambda_0\, ,\ \kappa_0\, ,\ h_{t0}\, ,\ \frac{A_0}{M_{1/2}}\, ,\ \frac{m_0^2}{M_{1/2}^2}, \tag{4.2}$$

where A_0, $M_{1/2}$ and m_0^2 are the universal trilinar coupling, gaugino mass and scalar mass respectively. In order to scan the 5-dimensional parameter space of the C$(M+1)$SSM, we proceed as in [7,9].

First, we scan over the GUT scale parameters (4.2) and integrate numerically the renormalization group equations [5] down to the SUSY scale in each case.

Then, we minimize the complete two loop effective potential in order to obtain the Higgs VEVs h_1, h_2, s. In principle, we could have followed the same procedure as in Sect. 3 to obtain the dominant two loop corrections, i.e. replacing h_1 by h_1' and h_t by $h_t(m_t)$. However, in order to obtain numerically correct results also in the regime $M_{\rm SUSY} < 1\,$TeV, we did not expand $V^{(1)}$ in powers of $h_i/M_{\rm SUSY}$, i.e. we used the full expression (2.3) for $V^{(1)}$. Then it becomes inconvenient to perform the field redefinition (3.6), which is implicitly non-linear due to the h_1 dependence of t via m_t. Therefore we proceed differently: For a given set of low energy parameters, which are implicitly obtained at the scale $Q \sim M_{\rm SUSY}$, we minimize directly

$$V_{\rm eff} = V^{(0)} + V^{(1)} + V^{(2)}, \tag{4.3}$$

with $V^{(0)}$ as in (2.2), $V^{(1)}$ as in (2.3) and $V^{(2)}$ as in (2.12). Points in the parameter space leading to deeper unphysical minima of the effective potential with $h_i = 0$ or $s = 0$ are removed.

The overall scale is then fixed by relating the VEVs h_i to the physical Z mass through

$$M_Z^2 = \frac{1}{2}\,(g_1^2 + g_2^2)(Z_{H_1}^2 h_1^2 + h_2^2), \tag{4.4}$$

with Z_{H_1} as in (2.13). Next, we throw away all points in the parameter space where the top quark mass (including corrections (2.14) to h_t) does not correspond to the measured $m_t^{\rm pole} = 173.8 \pm 5.2\,$GeV. We also ask for sfermions with masses $m_{\widetilde{f}} \gtrsim M_Z/2$ and gluinos with masses $m_{\widetilde{g}} \gtrsim 200\,$GeV.

Finally, the correct 3×3 Higgs mass matrix is related to the matrix of second derivatives of the Higgs potential at the minimum after dividing $(1/2)\partial_{h_1}^2 V_{\rm eff}$ by Z_{H_1}, and $(1/2)\partial_{h_1}\partial_{h_2} V_{\rm eff}$ and $(1/2)\partial_{h_1}\partial_s V_{\rm eff}$ by $Z_{H_1}^{1/2}$. For each point in the parameter space, we then obtain the two loop Higgs boson masses and couplings to gauge bosons. Then, we apply present constraints from the negative Higgs search at LEP (cf. Sect. 5 for details).

The results in Sect. 5 are based on scannings over $\sim 10^6$ points in the parameter space. The essential effect of all constraints within the C$(M+1)$SSM is to further reduce the allowed range for the Yukawa coupling λ to $\lambda \lesssim .3$.

4.2 General $(M+1)$SSM

In the general $(M+1)$SSM, we only assume that we are in a *local* minimum of the effective potential (4.3) and the running Yukawa couplings λ, κ, h_t are free of Landau singularities below the GUT scale. In order to scan the high dimensional parameter space (4.1) of the general $(M+1)$SSM we proceed as follows.

First, we use the three minimization equations of the full effective potential (4.3) with respect to h_1, h_2 and s in order to eliminate the parameters $m_{H_1}^2$, $m_{H_2}^2$ and m_S^2 in favor of the three Higgs VEVs. Using the relation (4.4), we replace h_1, h_2 by $\tan\beta$ and M_Z. Finally, (2.14), (2.15) and (2.16) allow us to express h_t in terms of $m_t^{\rm pole}$ and the other parameters.

We are then left with six "tree level" parameters λ, κ, A_λ, A_κ, s, $\tan\beta$, and three parameters appearing only through the radiative corrections, which we choose as $\widetilde{A}_t$, $M_{\rm SUSY}$ and δ, as defined in (2.6) and (2.8).

Requiring that the Yukawa couplings are free of Landau singularities below the GUT scale and using the renormalization group equations of the $(M+1)$SSM [5], one obtains upper limits on λ, κ, h_t at the SUSY scale. The latter turns into a lower bound on $\tan\beta$ depending mainly on $m_t^{\rm pole}$ and $M_{\rm SUSY}$. As expected from (3.11), we observe

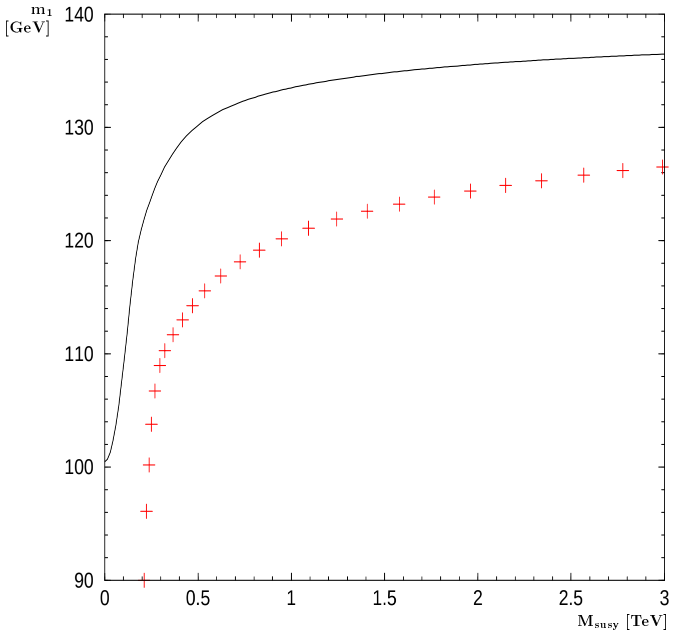

Fig. 1. Upper limits on the mass m_1 of the lightest CP even Higgs boson versus M_{SUSY} in the general $(M+1)$SSM (*straight line*) and the C$(M+1)$SSM (*crosses*)

that upper limits on Higgs masses are obtained when λ is maximal. From the renormalization group equations, one finds that the upper limit on λ increases with decreasing κ; thus we choose $\kappa \sim 0$ and $\lambda = \lambda_{\max} \sim .7$ (which still depends on h_t, i.e. on $\tan\beta$).

As already mentioned, one can see from (3.11) that the lightest Higgs mass is maximized for moderate values of $\tan\beta$. Hence, except in Fig. 4 where $\tan\beta$ varies, we fix $\tan\beta = 2.7$ which, as we shall see, maximizes the Higgs masses for $m_t^{\mathrm{pole}} = 173.8\,\mathrm{GeV}$.

Unless stated otherwise, the upper limits on the Higgs masses presented in the next section are given in the maximal mixing scenario ($\widetilde{A}_t = 6^{1/2} M_{\mathrm{SUSY}}$). We have also found that Higgs masses are maximized for small values of δ and fixed $\delta = 0$ (thus $m_Q = m_T = M_{\mathrm{SUSY}}$). In order to obtain the results presented in the next section, we have used numerical routines to maximize the Higgs masses with respect to the remaining three parameters A_λ, A_κ, s.

5 Reduced couplings versus mass bounds

Let us start with the mass m_1 of the lightest Higgs scalar, independently of its coupling to gauge bosons. The upper limit on m_1 in the general $(M+1)$SSM is plotted in Fig. 1 (straight line) as a function of M_{SUSY} (for $m_t^{\mathrm{pole}} = 173.8\,\mathrm{GeV}$). This limit is well above the one of the MSSM because of the additional tree level contribution to m_1^2 proportional to $\lambda^2 M_Z^2$ (cf. (1.3)). At $M_{\mathrm{SUSY}} = 1\,\mathrm{TeV}$ we have $m_1 \leq 133.5\,\mathrm{GeV}$ (in agreement with the analytic approximation (3.11)); at $M_{\mathrm{SUSY}} = 3\,\mathrm{TeV}$ this upper limit increases only by $\sim 3\,\mathrm{GeV}$. This weak dependence on M_{SUSY} is due to the negative two loop contributions to m_1.

Within the C$(M+1)$SSM, the combined constraints on the parameter space require λ to be small, $\lambda \lesssim .3$ [7, 9]. Accordingly, the upper limit on m_1 is very close to the one of the MSSM. It is shown as crosses in Fig. 1, and reaches 120 GeV at $M_{\mathrm{SUSY}} = 1\,\mathrm{TeV}$. In the following, we shall assume $M_{\mathrm{SUSY}} = 1\,\mathrm{TeV}$.

A parenthetical remark on the behavior for small M_{SUSY} is in order. From (2.7), it is obvious that, in the assumed limit $\delta \to 0$, the assumption of maximal stop mixing ($\widetilde{A}_t = 6^{1/2} M_{\mathrm{SUSY}}$) cannot be maintained for

$$\frac{\sqrt{6}-\sqrt{2}}{2} m_t < M_{\mathrm{SUSY}} < \frac{\sqrt{6}+\sqrt{2}}{2} m_t, \qquad (5.1)$$

because it would imply a negative stop mass squared. Therefore, in the general $(M+1)$SSM, we choose $\widetilde{A}_t$ in this regime such that the lightest stop mass squared remains positive. On the other hand, within the C$(M+1)$SSM, where soft SUSY breaking terms are related, the limit M_{SUSY} small is not feasable since it would contradict the negative results on sparticle searches.

As discussed in the introduction, the upper limit on m_1 is not necessarily physically relevant, since the coupling of the lightest Higgs to the Z boson can be very small. Actually, this phenomenon can also appear in the MSSM, if $\sin^2(\beta-\alpha)$ is small. However, the CP odd Higgs boson A is then necessarily light ($m_A \sim m_h < M_Z$ at tree level), and the process $Z \to hA$ can be used to cover this region of the parameter space in the MSSM. In the $(M+1)$SSM, a small gauge boson coupling of the lightest Higgs S_1 is usually related to a large gauge singlet component, in which case no (strongly coupled) light CP odd Higgs boson is available. Hence, Higgs searches in the $(M+1)$SSM have possibly to rely on the search for the second lightest Higgs scalar S_2.

Let us now define R_i as the square of the coupling ZZS_i divided by the corresponding standard model Higgs coupling:

$$R_i = (S_{i1}\sin\beta + S_{i2}\cos\beta)^2, \qquad (5.2)$$

where S_{i1}, S_{i2} are the H_1, H_2 components of the CP even Higgs boson S_i, respectively. Evidently, we have $0 \leq R_i \leq 1$ and unitarity implies

$$\sum_{i=1}^{3} R_i = 1. \qquad (5.3)$$

Fortunately, as was already mentioned, in the extreme case $R_1 \to 0$ the upper limit on m_2 is the same as the above upper limit on m_1. On the other hand, scenarios with, e.g., $R_1 \sim R_2 \sim 1/2$ are possible. In the following we will discuss these situations in detail.

We are interested in upper limits on the two lightest CP even Higgs bosons $S_{1,2}$. These are obtained in the limit where the third Higgs, S_3, is heavy and decouples, i.e. $R_3 \sim 0$. (This is the equivalent of the so-called decoupling limit in the MSSM: the upper bound on the lightest Higgs h is saturated when the second Higgs H is heavy and decouples.) Hence, we have $R_1 + R_2 \simeq 1$.

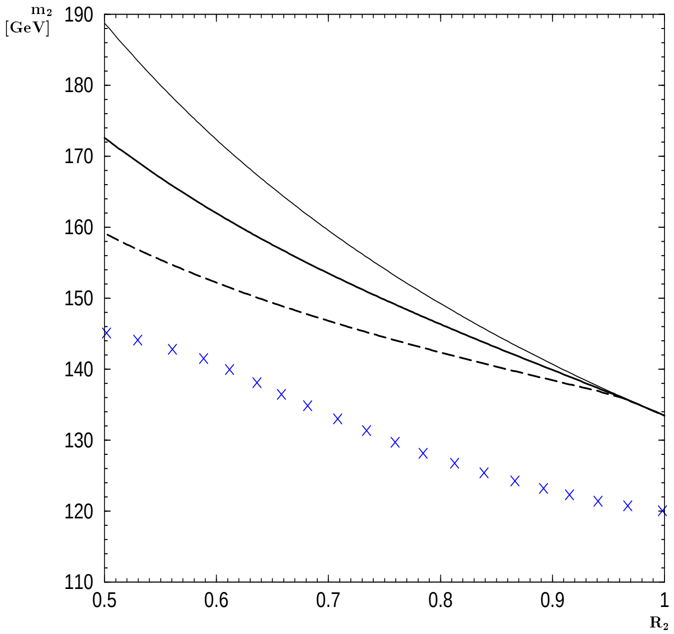

Fig. 2. Upper limits on the mass m_2 of the second lightest CP even Higgs (in the regime $R_2 > 1/2$) against R_2 in the general $(M+1)$SSM (*thin straight line*); the general $(M+1)$SSM with LEP constraints (5.4) (*thick straight line*); the general $(M+1)$SSM with expected LEP2 constraints (5.5) (*thick dashed line*); the C$(M+1)$SSM with LEP constraints (5.4) (*crosses*)

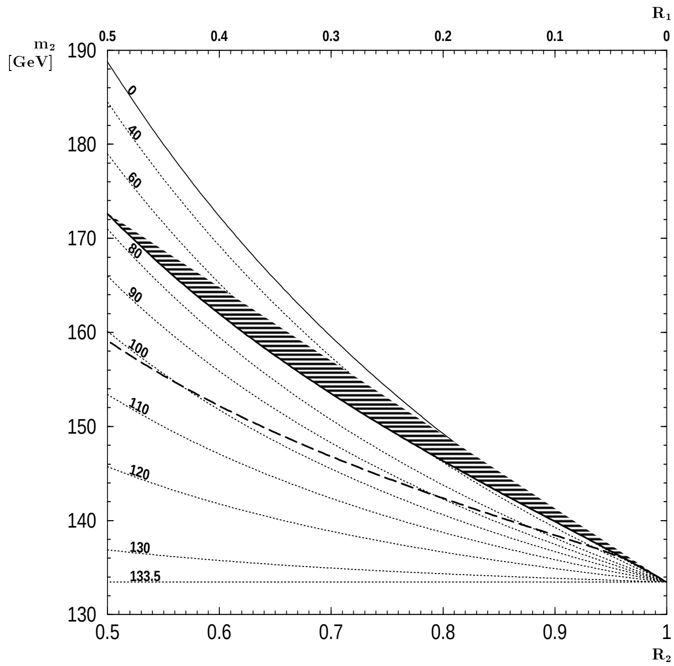

Fig. 3. Upper limits on the mass m_2 against R_2, for different lower limits on the mass m_1 (as indicated on each line in GeV) of the lightest Higgs boson for $1/2 < R_2 < 1$. $R_1 = 1 - R_2$ is shown on the top axis. The boundary of the shaded area corresponds to the thick line in Fig. 2; also the dashed line is the same as in Fig. 2

In the regime $R_1 \geq 1/2$ experiments will evidently first discover the lightest Higgs (with $m_1 \leq 133.5\,\mathrm{GeV}$ for $M_{\mathrm{SUSY}} = 1\,\mathrm{TeV}$). The "worst case scenario" in this regime corresponds to $m_1 \simeq 133.5\,\mathrm{GeV}$, $R_1 \simeq 1/2$; the presence of a Higgs boson with these properties has to be excluded in order to test this part of the parameter space of the general $(M+1)$SSM.

The regime where $R_1 < 1/2$ (and hence $1/2 < R_2 \leq 1$) is more delicate: here the lightest Higgs may escape detection because of its small coupling, and it may be easier to detect the second lightest Higgs. In Fig. 2 we show the upper limit on m_2 as a function of R_2 in the general $(M+1)$SSM as a thin straight line. For $R_2 \to 1$ (corresponding to $R_1 \to 0$) we obtain the announced result: the upper limit on a Higgs boson with $R \to 1$ is always given by the previous upper limit on m_1, even if the corresponding Higgs boson is actually the second lightest one. The same applies, of course, to the C$(M+1)$SSM where the upper limit on m_2 is also indicated as crosses in Fig. 2. In the following we will discuss this "delicate" regime, $R_1 < 1/2$ and $1/2 < R_2 \leq 1$, in some detail.

Fortunately, one finds that the upper limit on m_2 is saturated only when the mass m_1 of the lightest Higgs boson tends to 0. Clearly, one has to take into account the constraints from Higgs boson searches which apply to reduced couplings $R < 1/2$ – i.e. lower limits on m_1 as a function of $R_1 \simeq 1 - R_2$ – in order to obtain realistic upper limits on m_2 versus R_2.

Lower limits on m_1 as a function of R_1 (in the regime $R_1 < 1/2$) have been obtained at LEP [15]. We use the following analytic approximation for the constraints on R_1 versus m_1 in this regime:

$$\log_{10} R_1 < \frac{m_1}{45\,\mathrm{GeV}} - 2. \tag{5.4}$$

The resulting upper limit on m_2 is shown in Fig. 2 as a thick straight line. This constraint is automatically included in the C$(M+1)$SSM results (crosses). Present and future Higgs searches at LEP will lead to more stringent constraints in the regime $.1 < R_1 < 1/2$ [16]. We approximate the possible constraints from a run at 198 GeV c.m. energy and $200\,\mathrm{pb}^{-1}$ by

$$\ln R_1 < 2\left(\frac{m_1}{98\,\mathrm{GeV}}\right)^4 - 3. \tag{5.5}$$

The resulting upper limit on m_2 is shown in Fig. 2 as a thick dashed line.

It would be desirable to have the upper limit on m_2 in the general $(M+1)$SSM for arbitrary lower limits on m_1 as a function of R_1. To this end we have produced Fig. 3. The different dotted curves show the upper limit on m_2 as a function of R_2 for different lower limits on m_1 (as indicated on each curve) as a function of R_1 (as indicated at the top of Fig. 3).

In practice, Fig. 3 can be used to obtain upper limits on the mass m_2, in the regime $R_1 < 1/2$, for arbitrary experimental lower limits on the mass m_1: For each value of the coupling R_1, which corresponds to a vertical line in Fig. 3, one has to find the point where this vertical line crosses the dotted curve associated to the corresponding

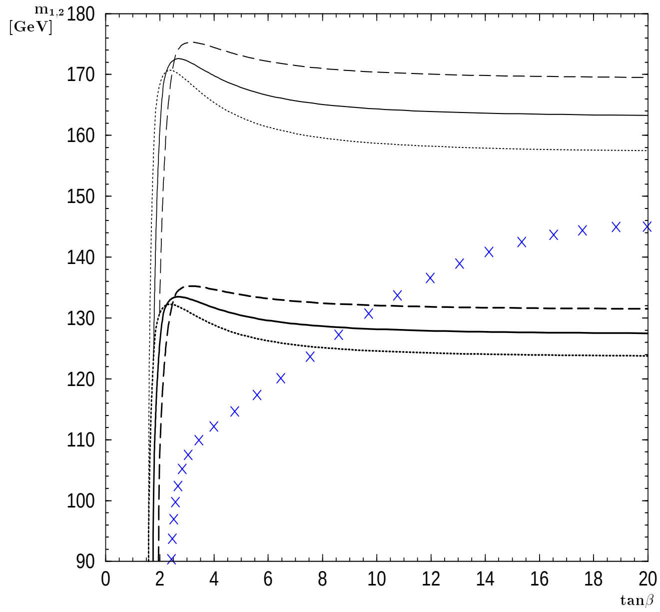

Fig. 4. Upper limits on $m_{1,2}$ with $R_{1,2} = 1$ (*thick lines*), and upper limits on m_2 with $R_2 = 1/2$ (*thin lines*) versus $\tan\beta$ in the general $(M+1)$SSM for $m_t^{\mathrm{pole}} = 173.8\,\mathrm{GeV}$ (straight), 179 GeV (*dashed*) and 168.6 GeV (*dotted*); upper limit on m_2 in the C$(M+1)$SSM (*crosses*) for $m_t^{\mathrm{pole}} = 173.8 \pm 5.2\,\mathrm{GeV}$. The LEP constraints (5.4) are taken into account in each case

experimental lower limit on m_1. Joining these points by a curve leads to the upper limit on m_2 as a function of R_2. We have indicated again the present LEP limit (5.4), already shown in Fig. 2, which excludes the shaded region ($m_2 > 172.5\,\mathrm{GeV}$ for $R_2 = .5$, $m_2 > 150\,\mathrm{GeV}$ for $R_2 = .75$ et cetera). We have also shown again the possible LEP2 constraints on m_2 arising from (5.5) as a thick dashed line.

Lower experimental limits on a Higgs boson with $R > 1/2$ restrict the allowed regime for m_2 (for $R_2 > 1/2$) in Fig. 3 from below. The present lower limits on m_2 from LEP are not visible in Fig. 3, since we have only shown the range $m_2 > 130\,\mathrm{GeV}$. Possibly Higgs searches at the Run II of the Tevatron push the lower limits on m_2 upwards into this range. This would be necessary if one aims at an exclusion of the "delicate" regime of the $(M+1)$SSM: Then, lower limits on the mass m_2 – for any value of R_2 between 1/2 and 1 – of at least 133.5 GeV are required; the precise experimental lower limits on m_2 as a function of R_2, which would be needed to this end, will depend on the achieved lower limits on m_1 as a function of R_1 in the regime $R_1 < 1/2$.

In principle, from (5.3), one could have $R_2 > R_1$ with R_2 as small as 1/3. However, in the regime $1/3 < R_2 < 1/2$, the upper bound on m_2 as a function of R_2 for different fixed values of m_1 can only be saturated if $R_1 = R_2$. Then it is sufficient to look for a Higgs boson with a coupling $1/3 < R < 1/2$ and a mass $m \lesssim 133.5\,\mathrm{GeV}$ to cover this region of the parameter space of the $(M+1)$SSM.

Finally, we consider the dependence of the upper bounds on $m_{1,2}$ on $\tan\beta$ and the top quark pole mass. In Fig. 4 we plot the upper limit on $m_{1,2}$ (for $R_{1,2} = 1$) against $\tan\beta$ for $m_t^{\mathrm{pole}} = 173.8\,\mathrm{GeV}$ as a thick straight line. Remarkably, as announced before, this $\tan\beta$ dependence is very different from the MSSM: the maximum is assumed for $\tan\beta \simeq 2.7$ (with $m_{1,2} \simeq 133.5\,\mathrm{GeV}$ in agreement with Figs. 2 and 3). The origin of this $\tan\beta$ dependence is the tree level contribution $\sim \lambda^2 \sin^2\beta$ to (3.12). The height and the location of the maximum varies somewhat with m_t^{pole}; the thick dashed and dotted curves correspond to $m_t^{\mathrm{pole}} = 173.8 \pm 5.2\,\mathrm{GeV}$, respectively. The absolute maximum is at $\tan\beta \simeq 3$ with $m_{1,2} \simeq 135\,\mathrm{GeV}$.

In the "delicate" regime, where one has to search for the second lightest Higgs with R_2 between 1/2 and 1, one could worry whether the $\tan\beta$ dependence of the upper limit on m_2 is different. This is not the case: as a thin straight line we show the upper limit on m_2 in the extreme case $R_2 = 1/2$ and $m_t^{\mathrm{pole}} = 173.8\,\mathrm{GeV}$ (where the LEP constraint (5.4) is taken into account), which assumes again its maximum for $\tan\beta \simeq 2.7$ (now with $m_2 \simeq 172.5\,\mathrm{GeV}$ in agreement with Figs. 2 and 3). As above, the thin dashed and dotted curves correspond to $m_t^{\mathrm{pole}} = 173.8 \pm 5.2\,\mathrm{GeV}$, respectively, and the absolute maximum is at $\tan\beta \simeq 3$ with $m_2 \simeq 175.5\,\mathrm{GeV}$. Within the C$(M+1)$SSM, where λ is small, the dependence of the upper limit on m_2 on $\tan\beta$ resembles more the one of the MSSM as shown as crosses in Fig. 4.

To conclude, we have studied the CP even Higgs sector of the general $(M+1)$SSM and the C$(M+1)$SSM including the dominant two loop corrections to the effective potential. We have emphasized the need to search for Higgs bosons with reduced couplings, which are possible within this model. Our main results are presented in Fig. 3, which allows one to obtain the constraints on the Higgs sector of the model both from searches for Higgs bosons with weak coupling ($R < 1/2$), and strong coupling ($R > 1/2$). The necessary (but not sufficient) condition for testing the complete parameter space of the $(M+1)$SSM is to rule out a CP even Higgs boson with a coupling $1/3 < R < 1$ and a mass below 135 GeV. The sufficient condition (i.e. the precise upper bound on m_2 versus R_2) depends on the achieved lower bound on the mass of a "weakly" coupled Higgs (with $0 < R < 1/2$) and can be obtained from Fig. 3. At the Tevatron this would probably require an integrated luminosity of up to $30\,\mathrm{fb}^{-1}$ [17]. If this cannot be achieved, and no Higgs is discovered, we will have to wait for the results of the LHC in order to see whether supersymmetry beyond the MSSM is realized in nature.

References

1. Y. Okada, M. Yamaguchi, T. Yanagida, Prog. Theor. Phys. **85**, 1 (1991); Phys. Lett. B **262**, 54 (1991); J. Ellis, G. Ridolfi, F. Zwirner, Phys. Lett. B **257**, 83 (1991); Phys. Lett. B **262**, 477 (1991); H. Haber, R. Hempfling, Phys. Rev. Lett. **66**, 1815 (1991); R. Barbieri, M. Frigeni, F. Caravaglios, Phys. Lett. B **258**, 167 (1991); A. Yamada, Phys. Lett. B **263**, 233 (1991); A. Brignole, J. Ellis, G. Ridolfi, F. Zwirner, Phys. Lett. B **271**, 167 (1991); D. Pierce, A. Papadopoulos, S. Johnson, Phys. Rev. Lett. **68**, 3678 (1992); P. Chankowski, S. Pokorski, J. Rosiek, Phys. Lett. B **274**, 191 (1992)

2. J. Espinosa, M. Quiros, Phys. Lett. B **266**, 389 (1991); J. Kodaira, Y. Yasui, S. Sasaki, Phys. Rev D **50**, 7035 (1994); J. Casas, J. Espinosa, M. Quiros, A. Riotto, Nucl. Phys. B **436**, 3 (1995); J. Casas, J. Espinosa, M. Quiros, Phys. Lett. B **342**, 171 (1995); H. Haber, R. Hempfling, A. Hoang, Z. Phys. C **75**, 539 (1997); S. Heinemeyer, W. Hollik, G. Weiglein, Phys. Lett. B **440**, 296 (1998); Phys. Rev D **58**, 091701 (1998); Eur. Phys. J. C **9**, 343 (1999); Phys. Lett. B **455**, 179 (1999)
3. R. Hempfling, A. Hoang, Phys. Lett. B **331**, 99 (1994); R.-J. Zhang, Phys. Lett. B **447**, 89 (1999)
4. M. Carena, J. Espinosa, M. Quiros, C. Wagner, Phys. Lett. B **355**, 209 (1995); M. Carena, M. Quiros, C. Wagner, Nucl. Phys. B **461**, 407 (1996)
5. J.-P. Derendinger, C.A. Savoy, Nucl. Phys. B **237**, 307 (1984); J. Ellis, J.F. Gunion, H.E. Haber, L. Roszkowski, F. Zwirner, Phys. Rev. D **39**, 844 (1989); M. Drees, Int. J. Mod. Phys. A **4**, 3635 (1989); P. Binetruy, C.A. Savoy, Phys. Lett. B **277**, 453 (1992); J. Espinosa, M. Quiros, Phys. Lett. B **279**, 92 (1992); Phys. Lett. B **302**, 51 (1993); T. Moroi, Y. Okada, Phys. Lett. B **295**, 73 (1992)
6. U. Ellwanger, M. Lindner, Phys. Lett. B **301**, 365 (1993); U. Ellwanger, Phys. Lett. B **303**, 271 (1993); T. Elliott, S.F. King, P.L. White, Phys. Lett. B **305**, 71 (1993); Phys. Lett. B **314**, 56 (1993); Phys. Rev. D **49**, 2435 (1994); P. Pandita, Phys. Lett. B **318**, 338 (1993), Z. Phys. C **59**, 575 (1993); S. Ham, S. Oh, B. Kim, J. Phys. G **22**, 1575 (1996)
7. U. Ellwanger, M. Rausch de Traubenberg, C.A. Savoy, Phys. Lett. B **315**, 331 (1993)
8. T. Elliott, S.F. King, P.L. White, Phys. Lett. B **351**, 213 (1995)
9. U. Ellwanger, M. Rausch de Traubenberg, C.A. Savoy, Nucl. Phys. B **492**, 21 (1997)
10. S.F. King, P.L. White, Phys. Rev. D **52**, 4183 (1995)
11. J. Kamoshita, Y. Okada, M. Tanaka, Phys. Lett. B **328**, 67 (1994); U. Ellwanger, M. Rausch de Traubenberg, C.A. Savoy, Z. Phys. C **67**, 665 (1995); F. Franke, H. Fraas, Phys. Lett. B **353**, 234 (1995); S.F. King, P.L. White, Phys. Rev. D **53**, 4049 (1996); S. Ham, S. Oh, B. Kim, Phys. Lett. B **414**, 305 (1997); N. Krasnikov, Mod. Phys. Lett. A **13**, 893 (1998)
12. U. Ellwanger, C. Hugonie, Eur. Phys. J. C **5**, 723 (1998) and hep-ph/9812427 (to appear in Eur. Phys. J.)
13. G. Yeghiyan, hep-ph/9904488
14. E. Accomando et al., Higgs Physics at LEP2, CERN Yellow Report CERN-96-01, hep-ph/9602250; ALEPH, DELPHI, L3 and OPAL Collaborations, CERN-EP/99-060
15. ALEPH Collaboration, Phys. Lett. B **313**, 312 (1993); Phys. Lett. B **412**, 173 (1997); Phys. Lett. B **440**, 419 (1998)
16. E. Gross, A. Read, D. Lellouch, CERN-EP/98-094, Proceedings 12th Les Rencontres de Physique de la Vallée d'Aoste (1998)
17. J. Conway, Higgs Searches in Run II at the Tevatron, Proceedings 13th Les Rencontres de Physique de la Vallée d'Aoste (1999); Physics at Tevatron Run II Workshop, Higgs Working Group Final Report (to appear)
18. C. Caso et al. (Particle Data Group), Eur. Phys. J. C **3**, 1 (1998)